

Adaptive Chirplet Transform and Deep Learning Algorithms for EEG based Sleep Stage Detection

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Contents

1	Introduction	1
2	Background and Literature Review	2
2.1	Introduction to EEG Analysis	2
2.2	Traditional Methods in EEG Analysis	2
2.2.1	Event-Related Potentials (ERPs)	2
2.2.2	Spectral Analysis	3
2.2.3	EEG Data Pre-processing, Filtering and Artifact Removal	3
2.3	Adaptive Chirplet Transform (ACT)	4
2.4	EEG and Deep Learning	4
2.4.1	Input to Neural Networks	5
2.4.2	Deep Learning Architectures	8
2.5	Summary of Research Gaps and Objectives	10
2.5.1	Identified Gaps	10
2.5.2	Objectives	10
3	Progress to Date	10
3.1	Window Choice	12
4	Future Work	12
4.1	Timeline	13

List of Figures

1	Artifact removal methods in Deep Learning EEG classification, as summarized in a review of 89 studies.	6
2	Deep learning architectures across 154 studies analyzed in "Deep learning-based electroencephalography analysis: a systematic review". 'N/M' stands for 'Not mentioned' and accounts for papers which have not reported the respective deep learning methodology aspect under analysis. (a) Architectures. (b) Distribution of architectures across years. (c) Distribution of input type according to the architecture category. (d) Distribution of number of neural network layers.	7
3	Deep learning architectures utilized in EEG sleep stage detection, as reviewed across 89 studies. The figure highlights the diversity of approaches, including CNNs, RNNs, LSTMs, and hybrid models, among others.	8
4	Gantt chart showing the timeline.	13

1 Introduction

Building on the diverse methodologies for EEG analysis, this thesis specifically focuses on the application of the Adaptive Chirplet Transform combined with deep learning to detect sleep stage. Sleep stage detection is a critical area of research due to its significance in diagnosing sleep disorders, understanding brain function during rest, and developing interventions to improve sleep quality. Deep learning offers a transformative approach by automating the extraction of meaningful features from complex, high-dimensional EEG data. Unlike traditional manual scoring methods or simpler classification techniques, deep learning models excel at capturing intricate temporal and spatial patterns within EEG signals. By leveraging publicly available datasets, this study aligns with broader scientific practices for benchmarking and comparability while avoiding the logistical and ethical complexities of collecting new EEG data.

Techniques such as the Fourier Transform and Wavelet Transform have traditionally dominated time-frequency analysis in EEG research. The Fourier Transform decomposes signals into their constituent frequencies, providing valuable insights into rhythmic brain activity. Wavelet Transforms extend this by combining frequency and temporal information, enabling the analysis of dynamic changes over time. However, these methods face limitations when capturing complex, non-stationary signal features, such as rapid frequency modulations or localized transient events. This paper adopts the Adaptive Chirplet Transform (ACT) as a promising alternative. [1] [2,3] Unlike the fixed basis functions of Fourier and Wavelet Transforms, the ACT dynamically adapts to the signal's local characteristics, offering a more flexible and precise representation of time-varying EEG patterns. These qualities make the ACT particularly suitable for sleep stage detection, where subtle physiological changes manifest as transient shifts in frequency content.

Despite its potential, the application of the ACT to EEG data remains under-explored, with only a few studies demonstrating its capabilities. [2, 4–12] This gap presents an opportunity to advance EEG research by integrating the ACT with state-of-the-art deep learning architectures. The ACT's ability to provide a detailed, dynamic representation of EEG signals enhances the input to deep learning models, potentially improving their classification accuracy and robustness. Sleep stage transitions, which involve nuanced changes in frequency and amplitude, are especially well-suited for analysis using the ACT.

The primary objective of this thesis is to address this research gap by employing the ACT as the feature extraction algorithm for the EEG data and feed it to several Deep Learning architectures for the application of sleep stage detection.

2 Background and Literature Review

2.1 Introduction to EEG Analysis

Electroencephalography (EEG) is a widely utilized technique that measures the electrical activity generated by the brain, allowing for brain wave visualization. Specifically, it detects minute differences in electric potential at the scalp, which result from the collective activity of post-synaptic potentials produced by neurons within the cortical layers.

EEG data is widely used to analyze brain activity, but EEG signals are highly complex due to their non-stationary nature, low signal to noise ratio, and the presence of artifacts from non-neural sources. Even after processing, the data persists being intricate and convoluted, making it difficult to interpret. To extract meaningful information from EEG signals, the data must undergo rigorous processing and analysis.

2.2 Traditional Methods in EEG Analysis

The analysis of EEG signals spans diverse methodologies, each suited to different applications and offering unique insights into brain activity.

2.2.1 Event-Related Potentials (ERPs)

Event-Related Potentials (ERPs) represent one of the most established approaches, involving the identification and analysis of brain responses that are time-locked to specific events, such as sensory stimuli or cognitive tasks. Examples include the P300, a positive deflection occurring roughly 300 ms after an expected stimulus, often used in BCIs and studies on attention and decision-making, and Steady-State Visual Evoked Potentials (SSVEPs), which leverage repetitive visual stimulation for applications like assistive technology and gaming. ERPs are widely used in cognitive neuroscience due to their ability to isolate specific neural processes. However, they require

significant manual preprocessing and are often limited by their reliance on predefined stimuli and rigid temporal windows, making them less adaptable to complex, real-world scenarios.

2.2.2 Spectral Analysis

Spectral analysis, another cornerstone of EEG research, focuses on the frequency content of brain signals to reveal insights into oscillatory activity across delta, theta, alpha, beta, and gamma bands. Each frequency band correlates with distinct cognitive and physiological states, such as delta waves linked to deep sleep or alpha waves associated with relaxation and attention. By employing time-frequency techniques like Wavelet Transforms or Chirplet Transforms, spectral analysis bridges the gap between traditional ERPs and modern deep learning by enabling the study of dynamic changes in oscillatory activity. It finds extensive use in both clinical and research settings, such as monitoring epilepsy, understanding attention dynamics, and tracking meditation progress. [13]

2.2.3 EEG Data Pre-processing, Filtering and Artifact Removal

Noise and artifacts from non-neural sources can be broken down into physiological noise and environmental noise. Physiological sources contributing the most noise come from eyeball movement, which is known as electrocorticogram (EOG), the cardiac signal, known as electrocardiogram (ECG), and muscular contractions, known as electromyography (EMG). Environmental noise can encompass electromagnetic fields in the vicinity caused by any other non biological sources, such as those caused from AC power lines or electronic devices in the room.

Signal processing methods are employed to address these challenges, enhancing the interpretability of EEG data. Signal Processing is a broad field and active area of research, and in the context of EEG signals it encompasses several different tasks, most notably pre-processing, feature extraction and analysis and interpretation of data. Pre-processing includes data filtering for noise reduction and artifact removal, segmenting the data into appropriate epochs and normalizing the data.

Filters such as low-pass, high-pass, band-pass, and notch filters are commonly used to retain relevant brainwave frequencies while suppressing noise from sources like eye movements or power line interference. By focusing on frequency-specific manipulations, data filtering provides a clean,

noise-reduced signal, forming the basis for subsequent, more sophisticated signal processing techniques.

Preprocessing EEG data is a critical step for the success of deep learning models. Filtering techniques include bandpass filtering to eliminate frequencies outside the range of interest (e.g., 0.5–50 Hz for most EEG tasks) and notch filtering to remove powerline interference such as 50 or 60 Hz noise. Artifact removal methods are employed to mitigate non-brain artifacts, utilizing techniques such as independent component analysis (ICA) to separate mixed signals and isolate artifacts, regression-based methods to subtract artifacts like eye blinks using reference signals, and consensus filtering, which applies multiple filtering methods to ensure signal integrity. Signal standardization, including normalization or z-scoring, ensures consistency and improves convergence during training. [13]

2.3 Adaptive Chirplet Transform (ACT)

The Adaptive Chirplet Transform (ACT) addresses these limitations by dynamically adapting to the local characteristics of signals. Unlike Fourier and Wavelet Transforms, the ACT provides a more flexible representation of time-varying EEG patterns. This makes it particularly suitable for capturing subtle frequency shifts, such as those observed during transitions between sleep stages.

2.4 EEG and Deep Learning

Analyzing EEG data manually is highly challenging due to its complex, non-stationary, and high-dimensional nature. Noise, artifacts, and inter-individual variability further complicate traditional analysis methods. Deep learning has emerged as a powerful tool for EEG analysis because it can automatically extract meaningful patterns and features from raw data without requiring extensive manual processing or domain-specific feature engineering. By leveraging its capacity to model intricate dependencies and handle large datasets, deep learning has enabled breakthroughs in tasks that were previously difficult to achieve using conventional methods. Furthermore, recent advances in deep learning architectures and computational resources have accelerated progress in this field.

Deep learning has been widely applied to various EEG classification tasks, which can be broadly categorized into domains such as sleep stage classification, mental state and emotion

recognition, disease detection and diagnosis, and brain-computer interfaces (BCIs). Sleep stage classification involves identifying distinct sleep stages, such as NREM and REM, based on EEG patterns, facilitating research in sleep medicine and disorders.

Mental state and emotion recognition tasks focus on classifying cognitive states (e.g., focused vs. relaxed) and emotions (e.g., happiness, stress), often utilized in human-computer interaction applications. Disease detection and diagnosis tasks aim to identify neurological disorders like epilepsy, Alzheimer’s disease, and Parkinson’s disease, with seizure detection from EEG signals being a prominent application. Meanwhile, BCIs enable control of external devices using EEG signals, involving tasks such as motor imagery classification and control signal generation. These applications showcase the versatility and transformative potential of deep learning in EEG analysis.

2.4.1 Input to Neural Networks

The choice of input data for neural networks is a critical factor in determining the performance of EEG analysis models. Different research approaches vary in their use of raw EEG signals, preprocessed data, and transformed representations to optimize neural network training.

While some studies utilize raw EEG signals as input relying simply on the feature extraction power of the Neural Network, the vast majority of research emphasizes preprocessing and filtering as essential steps. Techniques such as bandpass filtering, notch filtering to remove powerline interference, and independent component analysis (ICA) for ocular and muscular artifact removal are widely employed. These methods enhance signal quality and ensure cleaner inputs for neural networks. However, beyond initial preprocessing and filtering, there is little consensus among researchers on the extent of artifact removal or the specific techniques to employ. For instance, some studies apply rigorous artifact removal to eliminate non-neuronal signals entirely, while others retain certain artifacts to preserve additional features of the raw signal.

A recent review of 89 studies on deep learning for EEG classification highlights this lack of consensus and variability in pre-processing strategies. [14]The accompanying visual graph from the review illustrates the proportion of studies adopting different artifact removal methods, noting that the majority of studies either did not specify or did not use removal methods.

Transformations of EEG data into alternative domains further diversify input preparation strategies. Frequency-domain techniques, such as the Fourier Transform, decompose the signal into its

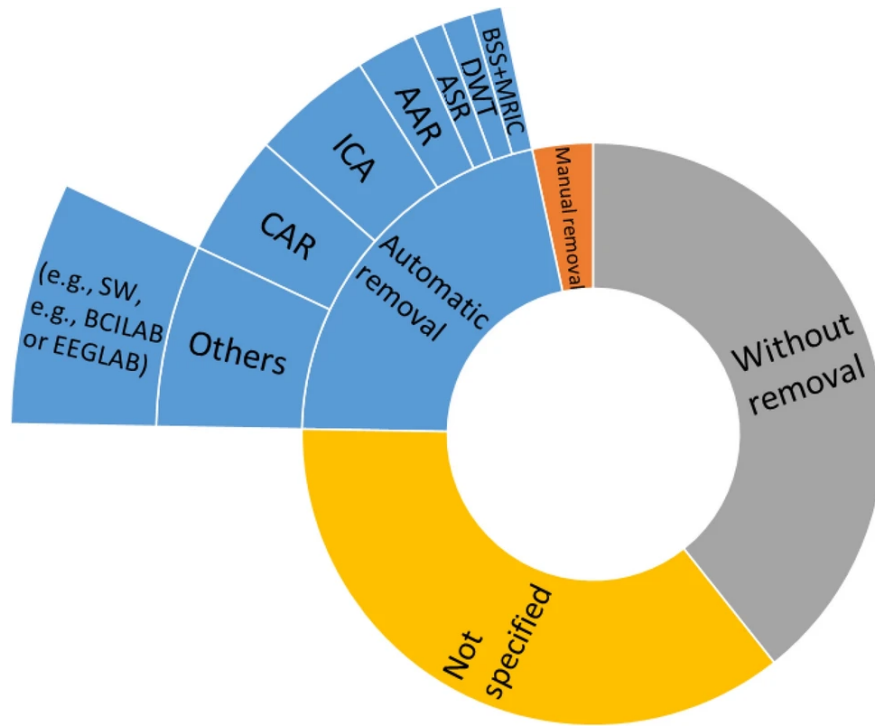


Figure 1: Artifact removal methods in Deep Learning EEG classification, as summarized in a review of 89 studies.

spectral components, providing insights into frequency-specific brain activity. The Wavelet Transform, offering a time-frequency representation, is frequently utilized to capture transient and non-stationary patterns in EEG signals. These transformed representations can augment neural network models by emphasizing specific aspects of brain activity relevant to the classification task. [15]

Figure 1 is complemented by another systematic review that analyzed 154 studies on deep learning-based EEG analysis. [16] This review explored different input strategies and neural network architectures, shedding light on trends across various methodologies. The insights from these reviews reinforce the importance of aligning input preparation strategies with specific research objectives.

This research will focus on using the Adaptive Chirplet Transform (ACT) as the chosen signal processing method due to its promising results in capturing time-frequency characteristics in other domains. The ACT's ability to provide detailed temporal and spectral representations aligns well with the goals of this study, offering a robust alternative to traditional signal processing and feature extraction techniques. By leveraging the ACT, the aim is to enhance the quality of inputs to neural networks and explore its potential to improve EEG and Deep Learning based sleep stage

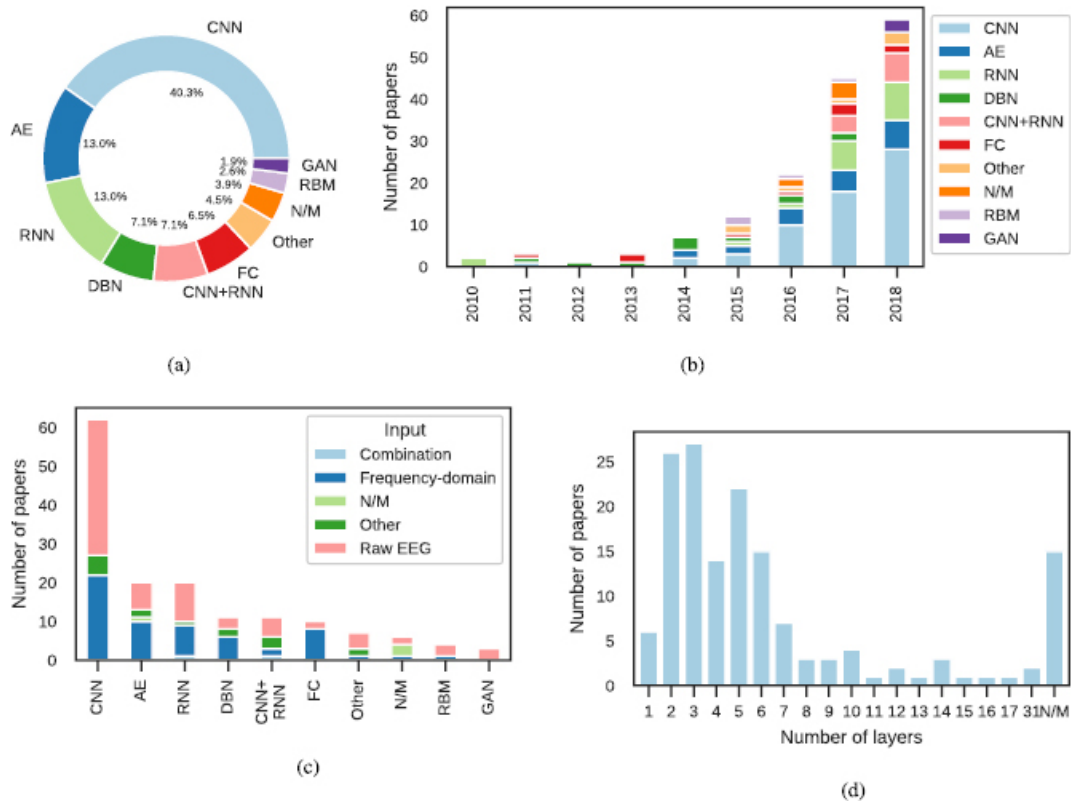


Figure 2: Deep learning architectures across 154 studies analyzed in "Deep learning-based electroencephalography analysis: a systematic review". 'N/M' stands for 'Not mentioned' and accounts for papers which have not reported the respective deep learning methodology aspect under analysis. (a) Architectures. (b) Distribution of architectures across years. (c) Distribution of input type according to the architecture category. (d) Distribution of number of neural network layers.

classification performance.

2.4.2 Deep Learning Architectures

Deep learning architectures play a crucial role in determining the performance and adaptability of models used for sleep stage detection with EEG data. The aforementioned review article of 89 studies on deep learning for EEG classification provides valuable insights into the variety of architectures employed in the field. [14] The findings are summarized in Figure 3, which illustrates the distribution of different deep learning architectures across these studies.

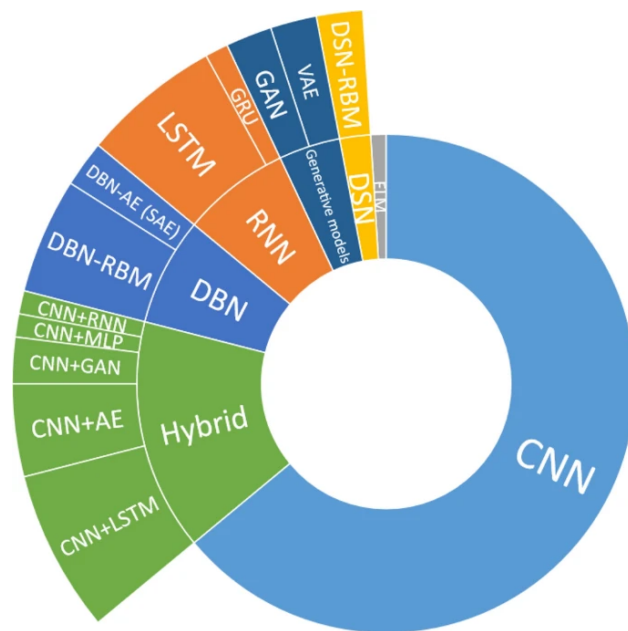


Figure 3: Deep learning architectures utilized in EEG sleep stage detection, as reviewed across 89 studies. The figure highlights the diversity of approaches, including CNNs, RNNs, LSTMs, and hybrid models, among others.

The landscape of deep learning applications for EEG-based sleep stage detection is marked by a variety of innovative approaches. Each architecture offers unique strengths and capabilities, reflecting the diverse challenges associated with analyzing EEG data. From capturing intricate spatial patterns to modeling temporal dynamics, the choice of architecture is often influenced by the specific requirements of the task, the nature of the data, and the desired level of interpretability. By exploring the relative merits and limitations of these architectures, researchers aim to uncover optimal strategies for improving classification performance and advancing the state of the field.

Convolutional Neural Networks (CNNs) are widely used for their ability to automatically extract spatial features from EEG data. [17] They excel in handling structured grid-like data and are particularly effective for time-series and spectrogram-based EEG representations. Many studies employed CNNs in pure form or hybridized with other architectures to enhance performance. Recurrent Neural Networks (RNNs), designed for sequential data, capture temporal dependencies in EEG signals and are particularly suited for tasks requiring the modeling of temporal dynamics, such as sleep stage transitions. Long Short-Term Memory Networks (LSTMs), a specialized type of RNN, address the vanishing gradient problem, enabling them to capture long-term dependencies in sequential data. [18] They are frequently used in hybrid models to complement the feature extraction capabilities of CNNs.

Deep Belief Networks (DBNs), comprising layers of stacked restricted Boltzmann machines, were among the earlier deep learning architectures applied to EEG data. Although less commonly used in recent years, they have demonstrated utility in unsupervised feature learning. Autoencoders are leveraged for dimensionality reduction and feature extraction, learning compact representations of EEG data, and are often used as a preprocessing step before classification. Hybrid models, which combine multiple architectures such as CNN-LSTM hybrids, aim to leverage the strengths of both spatial feature extraction and temporal modeling. These models are designed to capture both spatial and temporal dynamics of EEG signals, thereby improving classification accuracy.

While CNNs and their hybrid variations dominated the reviewed studies, the lack of consensus on the optimal architecture underscores the exploratory nature of this field. Moreover, it is important to note that this review was conducted prior to the advent of Transformers, a groundbreaking architecture that has revolutionized AI by excelling in capturing long-range dependencies and attention-based modeling. Transformers have already demonstrated immense potential in domains beyond EEG analysis, and their application to sleep stage detection represents a promising area for future research. [19]

The diversity of architectures highlighted in the review emphasizes the ongoing evolution of deep learning methodologies for EEG-based sleep stage detection. This study will explore how emerging architectures, such as Transformers, could further enhance the state-of-the-art in this field.

2.5 Summary of Research Gaps and Objectives

2.5.1 Identified Gaps

Despite advancements, the application of the ACT to EEG data remains limited, with few studies exploring its full potential. Similarly, the integration of advanced feature extraction methods with deep learning is still an emerging field.

There is a lack of studies integrating the ACT with deep learning for EEG analysis, particularly in sleep stage detection. The potential of the ACT for capturing subtle physiological changes has not been fully realized.

Furthermore there is no consensus on the optimal deep learning architecture for sleep stage detection. The field of Machine Learning is extremely active, with novel architectures being developed on a weekly basis.

2.5.2 Objectives

This research aims to evaluate the ACT's effectiveness in feature extraction and its integration with deep learning for sleep stage detection. Furthermore it aims to evaluate a variety of deep learning architectures to find which works best with the EEG signal transformed with the ACT as input and the sleep stage as the output.

3 Progress to Date

A Python implementation of the Adaptive Chirplet Transform (ACT) has been successfully developed. This implementation represents a crucial step forward, as the ACT is a central component of this study. To ensure the robustness and adaptability of the implementation, extensive testing was conducted across a wide range of parameter variations. This process resulted in the creation of a comprehensive library of mother chirplets, providing a critical resource for capturing the dynamic and non-stationary characteristics of EEG signals. The careful parameter optimization enhances the ACT's ability to adapt to local signal features, which is essential for analyzing transitions between sleep stages.

Another significant area of progress is the meticulous design of data segmentation and epoching

strategies. Recognizing that the accurate detection of sleep state transitions requires both temporal precision and computational feasibility, epoch lengths were carefully chosen. These lengths were optimized to strike a balance between capturing rapid changes in brain activity and maintaining manageable computational loads for the ACT and subsequent analyses. This alignment ensures that the data preparation phase supports the broader objective of precise sleep stage detection.

In parallel, a robust Python pipeline for EEG data preprocessing was developed. The preprocessing pipeline includes bandpass filtering to isolate relevant frequency bands associated with sleep stages and notch filtering to eliminate powerline interference, a common artifact in EEG data. These preprocessing steps are essential for ensuring that the EEG signals are free from noise and artifacts while retaining the physiological features necessary for accurate sleep stage classification. The pipeline was designed to process EEG data in a consistent and scalable manner, facilitating future experimentation with both small-scale and large-scale datasets.

To further strengthen the study, the Sleep-EDF Database was selected for analysis. [20] This dataset was chosen due to its high quality and relevance to sleep studies, providing a reliable foundation for the project's experiments. The selected dataset was subjected to the preprocessing pipeline, followed by the application of the ACT. Initial results from these analyses confirmed the pipeline's effectiveness, demonstrating its ability to generate time-frequency representations that capture the nuanced changes in EEG signals associated with sleep stage transitions. This validation step underscores the readiness of the developed methodology for subsequent phases of the research.

The development of the ACT implementation, the careful design of preprocessing and segmentation strategies, and the preliminary validation using real EEG data have collectively established a solid framework for the remaining phases of the study. These include feature extraction, integration with deep learning models, and the evaluation of various architectures for sleep stage classification. This progress provides confidence in the approach and positions the project for success in achieving its objectives.

3.1 Window Choice

Rectangular/Dirichlet vs Hanning vs Hamming (Cosine based) vs Blackman vs Bartlett
Bartlett is triangular Kaiser is Bessel functions Leakage consideration MNE allows for Hamming, Hanning and Blackman, Hamming default

Use a window with moderate main lobe width and low side lobes: Kaiser window with moderate (6 to 8). Blackman window if computational cost is not a concern. Hamming window for balanced performance and efficiency.

Try many?

Also Window size : Shorter windows provide better time resolution but poorer frequency resolution, and vice versa.

"FIR filters are easier to control, are always stable, have a well-defined passband, can be corrected to zero-phase without additional computations, and can be converted to minimum-phase. We therefore recommend FIR filters for most purposes in electrophysiological data analysis." [21]

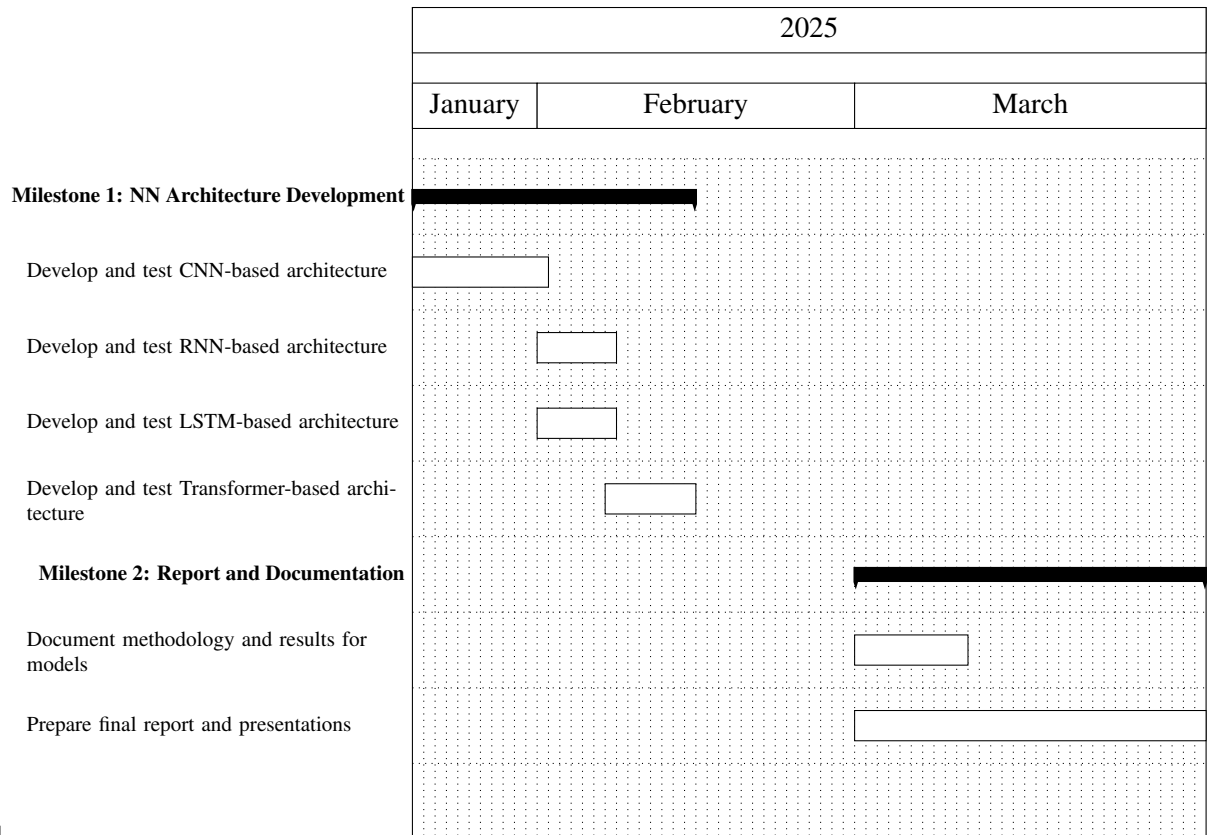
Hann - passband ripple 0.0545 dB stopband attenuation: 44 dB

Hamming 0.0194 dB 53 dB

4 Future Work

The remaining work for this project involves several key tasks to achieve the research objectives. The first step is the development and testing of machine learning architectures, starting with Convolutional Neural Networks (CNNs), followed by Recurrent Neural Networks (RNNs), Long Short-Term Memory (LSTM) networks, and Transformers. Each architecture will be implemented and evaluated for its performance in sleep stage detection using Adaptive Chirplet Transform (ACT)-based feature inputs. Once the architectures are developed, the next phase involves training these models on the Sleep-EDF Dataset, optimizing their hyperparameters, and comparing their performance based on standard metrics such as accuracy, F1-score, and precision-recall. Finally, the results will be documented, and a comprehensive report will be prepared to summarize the methodologies, findings, and implications of the study. This timeline ensures a structured approach to completing the project while addressing the identified research gaps.

4.1 Timeline



-01-01

Figure 4: Gantt chart showing the timeline.

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